

Mathematics Overview

The study of mathematics is of primary importance in developing skills in logical and analytical thinking and in applying mathematical treatments to problem solving. The attainment of such knowledge and skills is important and necessary for further study in mathematics and in disciplines which are mathematically based, such as physics, chemistry and engineering.

If algebra has been completed in the eighth-grade, mathematical study begins in the ninth grade with a year of geometry, one of the seven traditional liberal arts. Though the School has followed the Saxon math series in earlier grades, and though geometry is integrated into the Saxon series, the skills in logical thinking, reasoning, proofs and visualization merit a separate geometry course, for which we use the University of Chicago Mathematics Project's *Geometry*. In the tenth and eleventh grades, we use Saxon's *Algebra II* and *Advanced Mathematics*. Saxon mathematics is based on developing new skills and learning new concepts in small increments, providing continuing practice as new increments are added and testing students' progress with cumulative assessments. Calculus is studied in the twelfth grade, using Saxon's *Calculus with Trigonometry and Analytic Geometry*. This course does not specifically prepare the student for an Advanced Placement examination in calculus, although a well-prepared student may take the exam. A student preparing for the Advanced Placement exam might be better served by using Finney, Demana, Waits and Kennedy, as referred to in the calculus listing.

The mathematics and science curricula are coordinated in order to prepare the students mathematically for each stage of science study and to reinforce mathematical skills in science classes. The mathematics used in tenth grade physics will have been studied in eighth-grade algebra I and in ninth-grade geometry and, to some extent, concurrently with the study of algebra II in tenth grade. The mathematics needed for eleventh-grade chemistry and for twelfth-grade advanced biology will have been completed by the end of the study of algebra II in tenth grade. The astronomy/advanced physics elective in twelfth grade utilize the mathematics learned in eleventh-grade advanced mathematics and concurrently being learned in twelfth-grade calculus.

Algebra • Grade 9 (if not completed in Grade 8)

Required Text:

- Saxon, John H. *Algebra I: an Incremental Development*. 1990: Saxon Publishers.

SEMESTER ONE

- Addition and subtraction of fractions.
- Lines and segments
- Geometry review of perimeter and area
- Geometric shapes
- Volume
- Degree measurement
- Real Numbers and number line
- Multiplication and division of fractions
- Unit multiplier sets
- Surface area
- Absolute value
- Addition on the number line
- Rules for addition
- Definition of subtraction
- Opposites and multiple signs
- Rules for multiplication of signed numbers
- Inverse operations
- Rules for multiplication and division
- Division by zero
- Exchange of factors in multiplication
- Conversions of area and volume
- Reciprocal and multiplicative inverse
- Order of operations
- Symbols of inclusion
- Order of operations
- Multiple symbols of inclusion
- More on order of operations
- Products of signed numbers
- Evaluation of algebraic expression
- More complicated evaluations
- Terms and the distributive property
- Like terms
- Addition of like terms
- Exponents
- Powers of negative numbers
- Roots
- Evaluation of powers
- Product rule for exponents
- Like terms with exponents
- Statements with sentences
- Conditional equations
- Equivalent equations
- Additive property of equality
- Multiplicative property of equality
- Solution of equations
- More complicated equations
- More on the distributive property
- Simplifying decimal equations
- Fractional parts of numbers
- Negative exponents
- Zero exponents
- Decimal parts of a number
- Volume conversions
- Equations with parentheses
- Word problems
- Products of prime factors
- Statements about unequal quantities
- Greatest common factor
- Equations with parentheses
- Products of prime factors
- Greatest common factor
- Factoring the greatest common factor
- Negative exponents
- Graphing inequalities
- Trichotomy axiom
- Negated inequalities
- Graphing inequalities
- Two-step problems
- Solving multi-variable equations
- Least common multiple
- Addition of rational expressions
- Addition of abstract fractions
- Conjunctions
- Percentages less than 100
- Polynomials
- Addition and multiplication of polynomials
- Rectangular coordinates
- Graphs of linear equations
- Overall average
- Power rule for exponents
- Substitution axiom
- Dividing fractions
- Set notation
- Rearranging before subtraction subsets

SEMESTER TWO

- Rearranging before substitution
- Subsets
- Square roots
- Evaluation using plus or minus
- Product-of-squares rule
- Domain
- Additive property of equality
- Addition of radical expressions
- Weighted average
- Simplification of radical expressions
- Elimination
- More about complex fractions
- Factoring trinomials
- Trinomials with common factors
- Subscripted variables
- Factors that are sums
- Factoring the difference of two squares
- Scientific notation
- Closure
- Consecutive integers
- Consecutive odd and even integers
- Fraction and decimal word problems
- Rational equations
- Systems of equations with subscripted variables
- Operations with scientific notation
- Graphical solutions
- Writing the equation of a line
- Coin problems
- Multiplication of radicals
- Division of polynomials
- More on systems of equations
- More on division of polynomials
- Multiplication of radicals
- Division of polynomials
- More on systems of equations
- More on division of polynomials
- Solution of quadratic equations by factoring
- Value problems
- Slope-intercept method of graphing
- Word problems with two statements of equality
- Multiplicative property of equality
- Uniform-motion problems about equal distances
- Properties of the set of real numbers
- Rational expressions
- Uniform-motion problems of the form $D_1 + D_2 = N$
- Difference-of-two-squares theorem
- Pythagorean Theorem
- Distance between two points
- Algebraic proofs
- Uniform motion-unequal distances
- Square roots of large numbers
- Rounding numbers
- Square-root tables
- Factorable denominators
- Absolute-value inequalities
- Rational equations
- Abstract rational equations
- Equation of a line through two points
- Functions
- Functional notation
- Line parallel to a given line
- Equation of a line with a given slope
- Radical equations
- Slope formula
- Consistent, inconsistent, and dependent equations
- Conjunctions and disjunctions
- Multiplication of radical expressions
- Direct variation
- Inverse variation
- Exponential increases
- Linear inequalities
- Quotient rule for square roots
- Advanced trinomial factoring
- Factoring by Grouping
- Completing the square
- The quadratic formula
- Probability

Geometry • Grade 9 or 10

DESCRIPTION: Geometrical concepts and proofs of theorems are the foundations of the course; logical thinking is the primary objective. Drawing, visualizing, following algorithms, understanding properties and representing geometrical concepts with coordinates and networks are also emphasized.

METHOD: New lessons are introduced daily, with seven to nine lessons grouped into a chapter. Nightly homework includes problems covering both old and new concepts. Two quizzes and a test per chapter measure student progress and comprehension.

REQUIRED TEXT:

- Usiskin, Z., et al. *Geometry*, 2nd ed. Chicago, IL: The University of Chicago School Mathematics Project, 2002.

SUPPLEMENTAL TEXT:

- Euclid's *Elements*. Any edition.

SEMESTER ONE

- Chapter 1: Points and lines
- Chapter 2: Language and logic of geometry
- Chapter 3: Angles and lines
- Chapter 4: Reflections, translations and congruence
- Chapter 5: Proofs using congruence
- Chapter 6: Polygons and symmetry
- Chapter 7: Triangle congruence

SEMESTER TWO

- Chapter 8: Perimeters and areas
- Chapter 9: Three-dimensional figures
- Chapter 10: Surface areas and volumes
- Chapter 11: Indirect and coordinate proofs
- Chapter 12: Similarity
- Chapter 13: Similar triangles and trigonometry
- Chapter 14: Circles
- Euclid's *Elements*

Algebra II • Grade 10 or 11

DESCRIPTION: An integrated approach to mathematics is achieved through a review of basic algebra and geometry, followed by an incremental development of more advanced algebra, geometry, trigonometry, statistics and pre-calculus concepts.

METHOD: New concepts are introduced in each daily lesson. The lesson homework consists of thirty problems, most of which are review questions. The daily review of topics covered in previous classes and in previous lessons encourages retention of concepts through repetition and provides a natural segue into new concepts. Cumulative tests are taken after every fourth lesson.

REQUIRED TEXT AND MATERIALS:

- Saxon, John H. Jr. *Algebra II: An Incremental Development*, 2nd ed. Oklahoma City, OK: Thompson's School Book Depository, 1991.
- Any scientific calculator

SEMESTER ONE

- Lesson A: Geometry review; Angles; Review of absolute value; Properties and definitions
- Lesson B: Perimeter; Area; Volume; Surface area; Sectors of circles
- Lesson 1: Polygons; Triangles; Transversals; Proportional segments
- Lesson 2: Negative exponents; Product and power theorems for exponents; Circle relationships
- Lesson 3: Evaluation of expressions; Adding like terms
- Lesson 4: Distributive property; Solution of equations; Change sides—change signs
- Lesson 5: Word problems; Fractional parts of a number
- Lesson 6: Equations with decimal numbers; Consecutive integer word problems
- Lesson 7: Percent; Equations from geometry
- Lesson 8: Polynomials; Graphing linear equations; Intercept-slope method
- Lesson 9: Percent word problems
- Lesson 10: Pythagorean theorem
- Lesson 11: Addition of fractions; Inscribed angles
- Lesson 12: Equation of a line
- Lesson 13: Substitution; Area of an isosceles triangle
- Lesson 14: Equation of a line through two points; Equation of a line with a given slope
- Lesson 15: Elimination
- Lesson 16: Multiplication of polynomials; Division of polynomials
- Lesson 17: Subscripted variables; Angle relationships
- Lesson 18: Ratio word problems; Similar triangles
- Lesson 19: Value word problems; AA means AAA
- Lesson 20: Simplification of radicals; Line parallel to a given line
- Lesson 21: Scientific notation; Two statements of equality
- Lesson 22: Uniform motion problems—equal distances; Similar triangles and proportions
- Lesson 23: Graphical solutions

- Lesson 24: Fractional equations; Overlapping triangles
- Lesson 25: Monomial factoring; Cancellation; Parallel lines
- Lesson 26: Trinomial factoring; Overlapping right triangles
- Lesson 27: Rational expressions
- Lesson 28: Complex fractions; Rationalizing the denominator
- Lesson 29: Uniform motion problems: $D1 + D2 = k$
- Lesson 30: Deductive reasoning; Euclid; Vertical angles are equal; Corresponding interior and exterior angles; 180° in a triangle
- Lesson 31: Negative reciprocals; Perpendicular lines; Remote interior angles
- Lesson 32: Quotient theorem for square roots; Congruency; Congruent triangles
- Lesson 33: Major rules of algebra; Complex fractions
- Lesson 34: Uniform motion problems: $D1 + k = D2$
- Lesson 35: Angles in polygons; Inscribed quadrilaterals; Fractional exponents
- Lesson 36: Contrived problems; Multiplication of rational expressions; Division of rational expressions
- Lesson 37: Chemical compounds; Parallelograms
- Lesson 38: Powers of sums; Solving by factoring; Only zero equals zero
- Lesson 39: Difference of two squares; Parallelogram proof; Rhombus
- 12 Lesson 40: Abstract fractional equations
- Lesson 41: Units; Unit multipliers
- Lesson 42: Estimating with scientific notation
- Lesson 43: Sine, cosine and tangent; Inverse functions
- Lesson 44: Solving right triangles
- Lesson 45: Difference-of-two-squares theorem
- Lesson 46: More on radical expressions; Radicals to fractional exponents
- Lesson 47: Rate unit conversions; More on fractional exponents
- Lesson 48: Radical equations

- Lesson 49: Linear intercepts; Transversals
- Lesson 50: Quadratic equations; Completing the square
- Lesson 51: Imaginary numbers; Product-of-square-roots theorem; Euler's notation; Complex numbers
- Lesson 52: Chemical mixture problems
- Lesson 53: Metric unit conversions; English units to metric units; Weight combination by percent
- Lesson 54: Polar coordinates; Similar triangles
- Lesson 55: Advanced abstract equations; Word problems and quadratic equations
- Lesson 56: Angles in circles; Proofs
- Lesson 57: Ideal gas laws
- Lesson 58: Lead coefficients; More on completing the square
- Lesson 59: Experimental data; Simultaneous equations with fractions and decimals; Rectangular form to polar form
- Lesson 60: Direct and inverse variation
- Lesson 61: Chemical mixture problems, type B

SEMESTER TWO

- Lesson 62: Complex roots of quadratic equations
- Lesson 63: Addition of vectors
- Lesson 64: Complex fractions; Complex numbers
- Lesson 65: Advanced substitution
- Lesson 66: Signs of fractions; 30-60-90 triangles
- Lesson 67: Radical denominators
- Lesson 68: Scientific calculator; Scientific notation; Powers and roots
- Lesson 69: Gas law problems
- Lesson 70: Advanced abstract equations
- Lesson 71: Quadratic formula
- Lesson 72: Lines from experimental data; Negative angles
- Lesson 73: More on radical denominators
- Lesson 74: Uniform motion with both distances given
- Lesson 75: Factorable denominators and sign changes
- Lesson 76: Using both substitution and elimination; Negative vectors
- Lesson 77: Advanced radical equations; Multiple radicals
- Lesson 78: Force vectors at a point
- Lesson 79: Metric volume; 45-45-90 triangles
- Lesson 80: Direct and inverse variation as ratios
- Lesson 81: Complex numbers
- Lesson 82: Algebraic simplifications
- Lesson 83: Variable exponents
- Lesson 84: Solutions of equations
- Lesson 85: Systems of nonlinear equations
- Lesson 86: Greater than; Trichotomy and transitive axioms; Irrational roots

- Lesson 87: Slope formula
- Lesson 88: The distance formula; The relationship $PV = nRT$
- Lesson 89: Conjunctions; Disjunctions; Products of chords and secants
- Lesson 90: Systems of three equations
- Lesson 91: Linear inequalities; Greater than or equal to; Less than or equal to; Systems of linear inequalities
- Lesson 92: Boat-in-the-river problems
- Lesson 93: The discriminate
- Lesson 94: Dependent and independent variables; Functions; Functional notation
- Lesson 95: More nonlinear systems
- Lesson 96: Joint and combined variation; More on irrational roots
- Lesson 97: Advanced substitution
- Lesson 98: Relationships of numbers
- Lesson 99: Absolute value inequalities; Negative numbers and absolute value
- Lesson 100: Graphs of parabolas
- Lesson 101: Percent markups
- Lesson 102: Sums of functions; Products of functions
- Lesson 103: Advanced polynomial division
- Lesson 104: Complex numbers, rational numbers and decimal numerals
- Lesson 105: Advanced factoring
- Lesson 106: More on systems of three equations
- Lesson 107: Numbers, numerals and value; Number word problems
- Lesson 108: Sum and difference of two cubes
- Lesson 109: More on fractional exponents
- Lesson 110: Quadratic inequalities (greater than)
- Lesson 111: Three statements of equality
- Lesson 112: Quadratic inequalities (less than)

- Lesson 113: Logarithms; Antilogarithms
- Lesson 114: Nonlinear inequalities
- Lesson 115: Exponential equations; Exponential functions; Compound interest
- Lesson 116: Fundamental counting principle and permutations; Probability; Independent events
- Lesson 117: Letter symbols for sets; Set-builder notation
- Lesson 118: Logarithmic equations
- Lesson 119: Absolute value inequalities
- Lesson 120: Age word problems
- Lesson 121: Rational inequalities
- Lesson 122: Laws of logarithms; Intersection of sets; Union of sets; Venn diagrams
- Lesson 123: Locus; Basic construction
- Lesson 124: Conditions of congruence; Proofs of congruence; Isosceles triangles
- Lesson 125: Distance defined; Equidistance; Circle proofs
- Lesson 126: Rectangles; Squares; Isosceles trapezoids; Chords and arcs
- Lesson 127: Lines and planes in space
- Lesson 128: Circumscribed and inscribed; Inscribed triangles; Inscribed circles; Proof of the Pythagorean Theorem; Inscribed angles
- Lesson 129: Stem and leaf plots; Measures of central tendency; The normal curve; Standard deviation

Advanced Mathematics • Grade 11 or 12

DESCRIPTION: The topics covered in this course include a review of algebra and an in-depth presentation of trigonometry, logarithms, analytic geometry, and upper-level algebraic concepts. The study of geometry, begun in Algebra I, is also completed. In addition to teaching the concepts and skills necessary to succeed in calculus and in disciplines that are mathematically based (*e.g.* chemistry and physics), this course also requires students to work on problem-solving skills and to develop productive thought patterns.

METHOD: Students with good mathematical aptitudes will cover one lesson per class and thus the majority of the material in the text in one school year. Some students, however, may need two classes to cover each lesson, and thus require two school years to cover the same material. Each lesson is introduced first in class. Students then read the same lesson as homework before working a limited number of problems based on the new concept(s) they have learned. They also work problems based on previously learned concepts. During class, the teacher discusses with the students the solutions to the homework problems from the previous night before introducing the next lesson. Students are encouraged to demonstrate the solutions to problems at the blackboard. Cumulative tests are given after every fourth lesson. The tests emphasize material from four to eight lessons back, but also include problems from earlier lessons in order to ensure that students not forget foundational concepts.

REQUIRED TEXT:

- Saxon, John H. Jr. *Advanced Mathematics: An Incremental Development*, 2nd ed. Norman, OK: Saxon Publishers, 1998.

SUPPLEMENTAL TEXTS:

- Saxon, John H. Jr. *Advanced Mathematics: An Incremental Development, Solutions Manual*, 2nd ed. Norman, OK: Saxon Publishers, 1997.
- Saxon, John H. Jr. *Advanced Mathematics: An Incremental Development, Test Masters*, 2nd ed. Norman, OK: Saxon Publishers, 1997.

BACKGROUND TEXT RECOMMENDATIONS:

- Larson, Ron and Robert P. Hostetler. *Precalculus*, 5th ed. Boston: Houghton Mifflin, 2002.

SEMESTER ONE

- Lesson 1: Geometry review
- Lesson 2: More on area; Cylinders and prisms; Cones and pyramids; Spheres
- Lesson 3: Pythagorean theorem; Triangle inequalities (part 1); Similar polygons; Similar triangles
- Lesson 4: Construction
- Lesson 5: Exponents and radicals; Complex numbers; Areas of similar geometric figures; Diagonals of rectangular solids
- Lesson 6: Fractional equations; Radical equations; Systems of three linear equations
- Lesson 7: Inductive and deductive reasoning; Logic; The contra positive; Converse and inverse
- Lesson 8: Statements of similarity; Proportional segments; Angle bisectors and side ratios
- Lesson 9: Congruent figures; Proof outlines
- Lesson 10: Equation of a line; Rational denominators; Completing the square
- Lesson 11: Circles; Properties of circles; The quadratic formula
- Lesson 12: Angles and diagonals in polygons; Proof of the chord-tangent theorem
- Lesson 13: Intersecting secants; Intersecting secants and tangents; Products of chord segments; Products of secant and tangent segments
- Lesson 14: Sine, cosine and tangent; Angles of elevation and depression; Rectangular and polar coordinates; Coordinate conversion
- Lesson 15: Assumptions; Proofs
- Lesson 16: Complex fractions; Abstract equations; Division of polynomials
- Lesson 17: Proofs of the Pythagorean theorem; Proofs of similarity
- Lesson 18: Advanced word problems
- Lesson 19: Nonlinear systems; Factoring exponentials; Sum and difference of two cubes
- Lesson 20: Two special triangles
- Lesson 21: Evaluating functions; Domain and range; Types of functions; Tests for functions
- Lesson 22: Absolute value; Reciprocal functions
- Lesson 23: The exponential function; Sketching exponentials
- Lesson 24: Sums of trigonometric functions; Combining functions
- Lesson 25: Age problems; Rate problems

- Lesson 26: The logarithmic form of the exponential; Logarithmic equations
- Lesson 27: Related angles; Signs of trigonometric functions
- Lesson 28: Factorial notation; Abstract rate problems
- Lesson 29: The unit circle; Very large and very small fractions; Quadrantal angles
- Lesson 30: Addition of vectors; Overlapping triangles
- Lesson 31: Symmetry; Reflections; Translations
- Lesson 32: Inverse functions; Four quadrant signs; Inverse trigonometric functions
- Lesson 33: Quadrilaterals; Properties of parallelograms; Types of parallelograms; Conditions for parallelograms; Trapezoids
- Lesson 34: Summation notation; Linear regression; Decomposing functions
- Lesson 35: Change in coordinates; The name of a number; The distance formula
- Lesson 36: Angles greater than 360° ; Sums of trigonometric functions; Boat-in-the-river problems
- Lesson 37: The Line as a locus; The midpoint formula
- Lesson 38: Fundamental counting principle and permutations; Designated roots; Overall average rate
- Lesson 39: Radian measure of angles; Forms of linear equations
- Lesson 40: The argument in mathematics; The laws of logarithms; Properties of inverse functions
- Lesson 41: Reciprocal trigonometric functions; Permutation notation
- Lesson 42: Conic sections; Circles; Constants in exponential functions
- Lesson 43: Periodic functions; Graphs of $\sin D$ and $\cos D$
- Lesson 44: Abstract rate problems
- Lesson 45: Conditional permutations; Two-variable analysis using a graphing calculator
- Lesson 46: Complex roots; Factoring over the complex numbers
- Lesson 47: Vertical sinusoid translations; Arctan
- Lesson 48: Powers of trigonometric functions; Perpendicular bisectors
- Lesson 49: The logarithmic function; Development of the rules for logarithms
- Lesson 50: Trigonometric equations

- Lesson 51: Common logarithms and natural logarithms
- Lesson 52: The inviolable argument; Arguments in trigonometric equations
- Lesson 53: Review of unit multipliers; Angular velocity
- Lesson 54: Parabolas
- Lesson 55: Circular permutations; Distinguishable permutations
- Lesson 56: Triangular areas; Areas of segments; Systems of inequalities
- Lesson 57: Phase shifts in sinusoids; Period of a sinusoid
- Lesson 58: Distance from a point to a line; “Narrow” and “wide” parabolas
- Lesson 59: Advanced logarithm problems; The color of the White House
- Lesson 60: Factorable trigonometric equations; Loss of solutions caused by division
- Lesson 61: Single-variable analysis; The normal distribution; Box-and-whisker plots
- Lesson 62: Abstract coefficients; Linear variation
- Lesson 63: Circles and completing the square
- Lesson 64: The complex plane; Polar form of a complex number; Sums and products of complex numbers
- Lesson 65: Radicals in trigonometric equations; Graphs of logarithmic functions

SEMESTER TWO

- Lesson 66: Formulas for systems of equations; Phase shifts and period changes
- Lesson 67: Antilogarithms
- Lesson 68: Locus definition of a parabola; Translated parabolas; Applications; Derivation
- Lesson 69: Matrices; Determinants
- Lesson 70: Percentiles and z scores
- Lesson 71: The ellipse (part 1)
- Lesson 72: One side plus two other parts; Law of signs
- Lesson 73: Regular polygons
- Lesson 74: Cramer's rule
- Lesson 75: Combinations
- Lesson 76: Functions of $(-x)$; Functions of the other angle; Trigonometric identities (part 1); Rules of the game
- Lesson 77: Binomial expansions (part 1)
- Lesson 78: The hyperbola
- Lesson 79: DeMoivre's theorem; Roots of complex numbers
- Lesson 80: Trigonometric identities (part 2)
- Lesson 81: Law of cosines
- Lesson 82: Taking the logarithm of exponential equations
- Lesson 83: Simple probability; Independent events; Replacement
- Lesson 84: Factorable expressions; Sketching sinusoids
- Lesson 85: Advanced trigonometric equations; Clock problems
- Lesson 86: Arithmetic progressions and arithmetic means
- Lesson 87: Sum and difference identities; Tangent identities
- Lesson 88: Exponential functions (growth and decay)
- Lesson 89: The ellipse (part 2)
- Lesson 90: Double-angle identities; Half-angle identities

- Lesson 91: Geometric progressions
- Lesson 92: Probability of either; Notations for permutations and combinations
- Lesson 93: Advanced trigonometric identities; Triangle inequalities (part 2)
- Lesson 94: Graphs of secant and cosecant; Graphs of tangent and cotangent
- Lesson 95: Advanced complex roots
- Lesson 96: More double-angle identities; Triangle area formula; Proof of the law of sides; Equal angles imply proportional sides
- Lesson 97: The ambiguous case
- Lesson 98: Change of base; Contrived logarithm problems
- Lesson 99: Sequence notations; Advanced sequence problems; The arithmetic and geometric means
- Lesson 100: Product identities; More sum and difference identities
- Lesson 101: Zero determinants; 3x3 determinants; Determinant solutions of 3x3 systems; Independent equations
- Lesson 102: Binomial expansions (part 2)
- Lesson 103: Calculations with logarithms; Power of the hydrogen
- Lesson 104: Arithmetic series; Geometric series
- Lesson 105: Cofactors; Expansion by cofactors
- Lesson 106: Translations of conic sections; Equations of the ellipse; Equations of the hyperbola
- Lesson 107: Convergent geometric series
- Lesson 108: Matrix addition and multiplication
- Lesson 109: Rational numbers
- Lesson 110: Graphs of arcsine and arccosine; Graphs of arc secant and arc cosecant; Graphs of arctangent and arc cotangent
- Lesson 111: Logarithmic inequalities: Bases greater or less than one
- Lesson 112: Binomial theorem
- Lesson 113: Synthetic division; Zeros and roots
- Lesson 114: Graphs of factored polynomial functions
- Lesson 115: The remainder theorem

- Lesson 116: The region of interest
- Lesson 117: Prime and relatively prime numbers; Rational roots theorem
- Lesson 118: Roots of polynomial equations
- Lesson 119: Descartes' rule of signs; Upper and lower bound theorem; irrational roots
- Lesson 120: Matrix algebra; Finding inverse matrices
- Lesson 121: Piecewise functions; Greatest integer function
- Lesson 122: Graphs of rational functions; Graphs that contain holes
- Lesson 123: The general conic equation
- Lesson 124: Point of division formulas
- Lesson 125: Using the graphing calculator to graph; Solutions of systems of equations using the graphing calculator; Roots

Calculus • Grade 12

DESCRIPTION: This course presents and develops the concepts, methods and skills of calculus, with trigonometry and analytic geometry. It emphasizes problem solving and analytical thinking. The course begins with the study of the rate of change of functions, then studies derivatives and their applications, integration and applications of definite integrals, the calculus of transcendental functions (trigonometric, inverse trigonometric, exponential and logarithmic) and the techniques of integration.

METHOD: The teacher follows Saxon's method by introducing a new lesson in each class. As homework, students read the lesson explaining the same concepts introduced in class. They work some problems based upon these new concepts, and even more problems based upon concepts previously introduced. Before introducing the next lesson in class, the teacher discusses with the students the solutions to the homework problems from the previous night. Students are encouraged to demonstrate the solutions to problems at the blackboard. Tests are given after every fourth lesson. The tests emphasize material from four to eight lessons back, but also include problems from earlier lessons in order to ensure that students do not forget foundational concepts. The schedule of topics from Saxon's *Calculus* as they are listed below would fit an accelerated student who wished to attempt an Advanced Placement examination in calculus, although for that purpose the student might want to consider the text by Finney listed under the background text recommendations. The typical student will require three semesters to complete Saxon's *Calculus*. For two semesters, a realistic goal is to proceed at such a pace as to complete Lesson 104.

REQUIRED TEXT:

- Saxon, John H. Jr. and Frank Y.H. Wang. *Calculus with Trigonometry and Analytic Geometry*, 2nd ed. Norman, OK: Saxon, 2002.

SUPPLEMENTAL TEXTS:

- Saxon, John H. Jr. and Frank Y.H. Wang. *Calculus with Trigonometry and Analytic Geometry, Solutions Manual*, 2nd ed. Norman, OK: Saxon Publishers, Inc., 2002.
- Saxon, John H. Jr. *Advanced Mathematics: An Incremental Development, Test Masters*, 2nd ed. Norman, OK: Saxon Publishers, 1997.

BACKGROUND TEXT RECOMMENDATIONS:

- Finney, Ross L., et al. *Calculus, Graphical, Numerical, Algebraic*. Menlo Park, CA: Addison-Wesley, 1999.
- Thomas, George B. Jr. and Ross L. Finney. *Calculus and Analytic Geometry*, 8th ed. Menlo Park, CA: Addison-Wesley, 1992.

SEMESTER ONE

- Lesson 1: Real numbers; Review of algebraic manipulations
- Lesson 2: Review of equations of lines; Using the graphing calculator
- Lesson 3: The contra-positive, converse and inverse; If and only if statements
- Lesson 4: Radian measure of angles; Trigonometric ratios; The four quadrant signs; Simplifying trigonometric expressions
- Lesson 5: Review of word problems
- Lesson 6: Equations and graphs of functions; Functional notation; Domain and range
- Lesson 7: The unit circle; Centerline, amplitude and phase angle of sinusoids; Period of a function; Important numbers; Exponential functions
- Lesson 8: Pythagorean identities; Functions of $-$; Trigonometric identities; Co-functions; Similar triangles
- Lesson 9: Absolute value as a distance; Graphing “special” functions; Logarithms; Base 10 and base e ; Simple logarithm problems
- Lesson 10: Quadratic polynomials; Remainder theorem; Rational roots theorem
- Lesson 11: Continuity; Left- and right-hand limits
- Lesson 12: Sum and difference identities; Double-angle identities; Half-angle identities; Graphs of logarithmic functions
- Lesson 13: Inverse trigonometric functions; Trigonometric equations
- Lesson 14: Limit of a function
- Lesson 15: Interval notation; Products of linear factors; Tangents; Increasing and decreasing functions
- Lesson 16: Logarithms of products and quotients; Logarithms of powers; Exponential equations
- Lesson 17: Infinity as a limit; Undefined limits
- Lesson 18: Sums, differences, products and quotients of functions; Composition of functions
- Lesson 19: The derivative; Slopes of curves on a graphing calculator
- Lesson 20: Change of base; Graphing origin-centered conics on a graphing calculator
- Lesson 21: Translations of functions; Graphs of rational functions (part 1)
- Lesson 22: Binomial expansion; Recognizing the equations of conic sections
- Lesson 23: Trigonometric functions of $n\theta$; Graphing conics on a graphing calculator

- Lesson 24: New notation for the definition of the derivative; The derivative of x^n
- Lesson 25: The constant-multiple rule for derivatives; The derivatives of sums and differences; Proof of the derivative of a sum
- Lesson 26: Derivatives of e^x and $\ln |x|$; Derivatives of $\sin x$ and $\cos x$; Exponential growth and decay
- Lesson 27: Equation of the tangent line; Higher-order derivatives
- Lesson 28: Graphs of rational functions (part 2); A special limit
- Lesson 29: Newton and Leibniz; Differentials
- Lesson 30: Graph of $\tan \theta$; Graphs of reciprocal functions
- Lesson 31: Product rule; Proof of product rule
- Lesson 32: An anti-derivative; The indefinite integral
- Lesson 33: Factors of polynomial functions; Graphs of polynomial functions
- Lesson 34: Implicit differentiation
- Lesson 35: Integral of a constant; Integral of $kf(x)$; Integral of x^n
- Lesson 36: Critical numbers; A note about critical numbers
- Lesson 37: Differentiation by u substitution
- Lesson 38: Integral of a sum; Integral of $1/x$
- Lesson 39: Area under a curve; Upper and lower sums; Left, right and midpoint sums
- Lesson 40: Units for the derivative; Normal lines; Maximums and minimums on a graphing calculator
- Lesson 41: Graphs of rational functions (part 3)
- Lesson 42: The derivative of a quotient; Proof of the quotient rule
- Lesson 43: Area under a curve as an infinite summation
- Lesson 44: The chain rule; Alternate definition of the derivative; The symmetric derivative
- Lesson 45: Using f' to characterize f ; Using f' to find maximums and minimums
- Lesson 46: Related rates problems
- Lesson 47: Fundamental theorem of calculus (part 1); Riemann sums; The definite integral
- Lesson 48: Derivatives of trigonometric functions; Summary of rules for derivatives and differentials

- Lesson 49: Concavity and inflection points; Geometric meaning of the second derivative; First and second derivative tests
- Lesson 50: Derivatives of composite functions; Derivatives of products and quotients of composite functions
- Lesson 51: Integration by guessing
- Lesson 52: Maximization and minimization problems
- Lesson 53: Numerical integration of positive-valued functions on a graphing calculator
- Lesson 54: Velocity and acceleration; Motion due to gravity
- Lesson 55: Maclaurin polynomials
- Lesson 56: More integration by guessing
- Lesson 57: Properties of the definite integral
- Lesson 58: Explicit and implicit equations; Inverse functions
- Lesson 59: Computing areas; More numerical integration on a graphing calculator
- Lesson 60: Area between two curves; Area between curves using a graphing calculator
- Lesson 61: Playing games with f , f' and f''
- Lesson 62: Work, distance and rates
- Lesson 63: Critical number (closed interval) theorem
- Lesson 64: Derivatives of inverse trigonometric functions
- Lesson 65: Falling-body problems
- Lesson 66: u substitution; Change of variable; Proof of substitution theorem
- Lesson 67: Areas involving functions of y
- Lesson 68: Even and odd functions
- Lesson 69: Integration by parts (part 1)
- Lesson 70: Properties of limits; Some special limits
- Lesson 71: Solids of revolution (part 1): disks
- Lesson 72: Derivatives of a^x ; Derivatives of $\log_a x$; Derivative of $|f(x)|$

SEMESTER TWO

- Lesson 73: Integrals of ax ; Integrals of $\log x$
- Lesson 74: Fluid force
- Lesson 75: Continuity of functions
- Lesson 76: Integration of odd powers of $\sin x$ and $\cos x$
- Lesson 77: Pumping fluids
- Lesson 78: Particle motion (part 1)
- Lesson 79: L'Hôpital's rule
- Lesson 80: Asymptotes of rational functions
- Lesson 81: Solids of revolution (part 2): washers
- Lesson 82: Limits and continuity; Differentiability
- Lesson 83: Integration of even powers of $\sin x$ and $\cos x$
- Lesson 84: Logarithmic differentiation
- Lesson 85: The mean value theorem and its applications; Proof of Rolle's Theorem
- Lesson 86: Rules for even and odd functions
- Lesson 87: Solids of revolution (part 3): shells
- Lesson 88: Separable differential equations
- Lesson 89: Average value of a function; Mean value theorem for integrals and its proof
- Lesson 90: Particle motion (part 2)
- Lesson 91: Product and difference indeterminate forms
- Lesson 92: Derivatives of inverse functions
- Lesson 93: Newton's method
- Lesson 94: Solids of revolution (part 4): displaced axes of revolution
- Lesson 95: Trapezoidal rule; Error bound for the trapezoidal rule
- Lesson 96: Derivatives and integrals of functions involving absolute value
- Lesson 97: Solids defined by cross sections

- Lesson 98: Fundamental theorem of calculus (part 2); The natural logarithm function
- Lesson 99: Linear approximations using differentials
- Lesson 100: Integrals of powers of $\tan x$; Integrals of powers of $\cot x$; Integrals of $\sec x$ and $\csc x$
- Lesson 101: Limit of $(\sin x)/x$ for small x ; Proof of the derivative of $\sin x$
- Lesson 102: Derivatives of $\ln x$ and e^x
- Lesson 103: Proof of the fundamental theorem of calculus; Epsilon-delta proofs
- Lesson 104: Graphs of solutions of differential equations; Slope fields; Recognizing graphs of slope fields
- Lesson 105: Sequences; Limit of a sequence; Graphs of sequences; Characteristics of sequences
- Lesson 106: Introduction to parametric equations; Slope of parametric curves
- Lesson 107: Polar coordinates; Polar equations
- Lesson 108: Introduction to vectors; Arithmetic of vectors; Unit vectors and normal vectors
- Lesson 109: Arc length (part 1); Rectangular equations
- Lesson 110: Rose curves
- Lesson 111: The exponential indeterminate forms $0/0$, 1^∞ and $\infty/0$
- Lesson 112: Foundations of trigonometric substitution
- Lesson 113: Trigonometric substitution
- Lesson 114: Arc length (part 2): parametric equations
- Lesson 115: Partial fractions (part 1); Logistic differential equations
- Lesson 116: Series
- Lesson 117: Geometric series; Telescoping series
- Lesson 118: Limaçons and lemniscates
- Lesson 119: Parametric equations—second derivatives and tangent lines
- Lesson 120: Partial fractions (part 2)
- Lesson 121: Convergence and divergence; Series indexing; Arithmetic of series
- Lesson 122: Integration by parts (part 2)
- Lesson 123: Vector functions

- Lesson 124: Implicit differentiation (part 2)
- Lesson 125: Infinite limits of integration
- Lesson 126: Partial fractions (part 3)
- Lesson 127: P-series
- Lesson 128: Basic comparison test; Integral test; Proof of p-test
- Lesson 129: Area bounded by polar curves
- Lesson 130: Ratio test; Root test
- Lesson 131: Infinite integrands
- Lesson 132: Limit comparison test
- Lesson 133: Euler's method
- Lesson 134: Slopes of polar curves
- Lesson 135: Absolute convergence
- Lesson 136: Using the chain rule with the fundamental theorem of calculus